

Kinematics of reference frames in arbitrary spacetimes: facts and fiction

*Maurício O. Calvão
(IF-UFRJ)*



*The ways that can be walked
are not the eternal Way
The names that can be named
are not the eternal name*
[Laozi]



*As far as the laws of mathematics refer
to reality, they are not certain;
and as far as they are certain, they do
not refer to reality.*
[A. Einstein]



“The Dark Energy Revolution in Cosmology: Celebrating Ioav Waga’s Career”, IF-UFRJ (2018/09/28)

I. BACKGROUND

- What is a (relativistic) spacetime (ST)?

$$(\mathcal{M}, g_{ab}) \quad (-1, +1, \dots, +1)$$

- What is an instantaneous observer?

$$(\mathcal{E}, u^a|_{\mathcal{E}})$$

- What is an observer?

- a coordinate system or chart x^α
- a reference frame
- a future-pointing unit timelike (parametrized) curve $\mathcal{C}$



- **What is a reference frame?**
 - a coordinate system or chart
 - a congruence of observers $\Leftrightarrow u^a$ ✓
 - a tetrad field
 - a foliation of spacelike hypersurfaces
- **How is the kinematics of a reference frame described?**

$$\frac{(\delta\ell)^{\cdot}}{\delta\ell} = \frac{1}{3}\Theta + \sigma_{ab}n^an^b;$$

$$h_{ab}(n^b)^{\cdot} = (\omega_{ab} + \sigma_{ab} - \sigma_{cd}n^cn^dh_{ab})n^b$$

$$u_{a;b} = -a_a u_b + \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$



II. FACTS AND FICTION

- Can the Christoffel symbols, along an arbitrary observer, in an arbitrary spacetime, vanish?
 - Yes (Fermi coordinates) ✓
 - No
- Can the redshift, along an arbitrary null geodesic, in an arbitrary spacetime, vanish?
 - Yes (⇐ parallel transportation) ✓
 - No



- What is an inertial reference frame (IRF) in an arbitrary spacetime?

- a coordinate system/chart where $g_{\alpha\beta} = \eta_{\alpha\beta}$
- a reference frame whose kinematic parameters all vanish: $a_a = \Theta = \sigma_{ab} = \omega_{ab} = 0$
- a geodesic reference frame: $a^a = 0$
- a tetrad field
- a local tetrad



$$u_{a;b} = 0$$

- Is our definition of IRF vacuous?

Not at all: of course there are such frames in Minkowski spacetime!



- **But what about an arbitrary spacetime?**
 - *Ricci-flat case* (hint: Ricci identity):
only (locally isometric to) Minkowski ST!
 - *non-Ricci-flat case* (legitimately curved):

$$g_{\alpha\beta} = \text{diag} (b, A(x^i), A(x^i), A(x^i))$$

$$u^\alpha = \delta^a_0 / \sqrt{|b|}$$

$$R = \frac{3(A_x)^2 - 4AA_{xx} + 3(A_y)^2 - 4AA_{yy} + 3(A_z)^2 - 4AA_{zz}}{2A^3}$$



- **Generic reference frames in Minkowski ST**

- Is there a geodesic expanding RF?

Yes: Milne

- Is there an accelerated rigid non-rotating RF?

Yes: Rindler

- Is there an accelerated rigid rotating RF?

Yes: “rotating cylinder”

- Is there a geodesic rotating RF?

No!



- **Generic reference frames in arbitrary ST's**
 - Is there a geodesic RF?
Yes, of course
 - Is there a rigid RF?
No
 - Is there a geodesic non-rotating RF?
Yes (locally proper time synchronizable)



- “Expanding Universe”

- Is there a *nonexpanding* RF in $k = 0$ RW spacetime?

- Yes



- No

$$g_{\alpha\beta} = \text{diag}(-a^2(\eta), a^2(\eta), a^2(\eta), a^2(\eta))$$

$$u^\alpha = \left(\frac{1}{a^4}, \frac{\sqrt{1/a^6 - 1}}{\sqrt{3}a}, \frac{\sqrt{1/a^6 - 1}}{\sqrt{3}a}, \frac{\sqrt{1/a^6 - 1}}{\sqrt{3}a} \right)$$



- Is there a ***geodesic nonexpanding*** RF in $k = 0$ RW spacetime?

$N = 2$: **No!!**

$N \geq 3$: **???**

