

The Dark Energy Revolution in Cosmology

Celebrating Ioav Waga's Career



Effective search strategies for cosmic
topology in CMB maps

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Outline

1. Cosmic topology and detectability

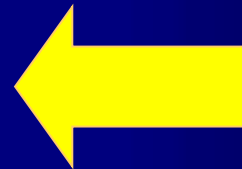
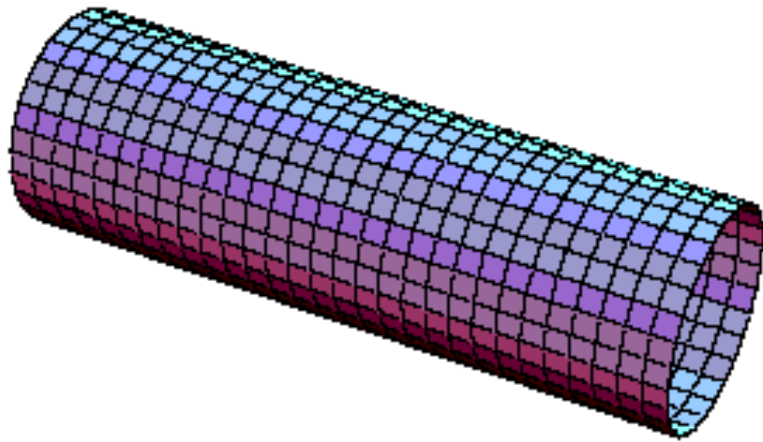
2. Circles in the sky - Detecting cosmic topology

3. Constraining observable parameters

4. Where to look for topology, and what it can tell us

The shape of the Universe

- GR is a local metric theory
- Topology is not fixed (but is constrained) by geometry
- No fundamental theory predicts topology
- Topology must be determined observationally!



$\mathbb{R}^2$ and $\mathbb{R}^1 \times S^1$: Same geometry, different topologies

Geometry and topology

Spacially homogeneous and isotropic Universe:

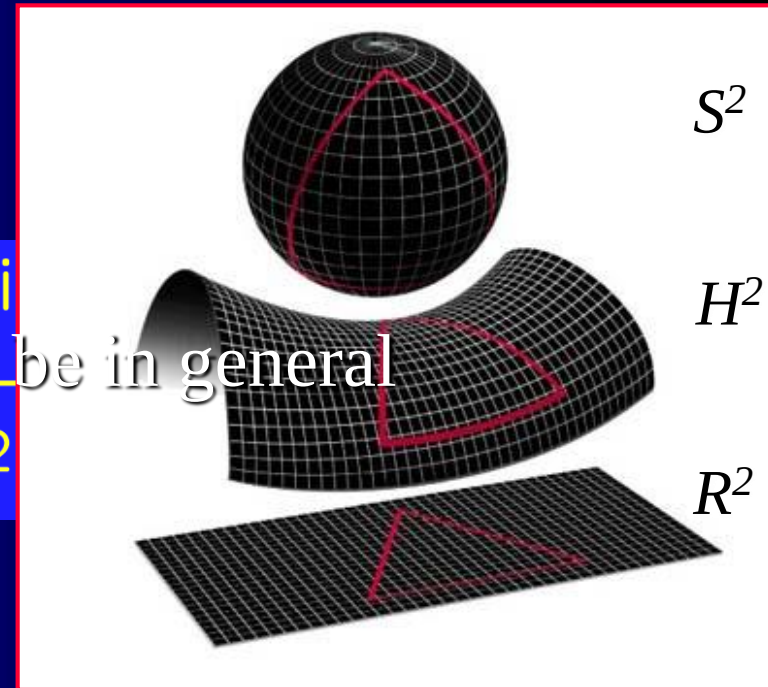
$$M_4 = \mathbb{R} \times M$$

Admits FLRW metric

$$ds^2 = c^2 dt^2 - a^2(t) d\ell^2$$

For $k = -1, 1, 0$, spatial sections will be in general quotient spaces

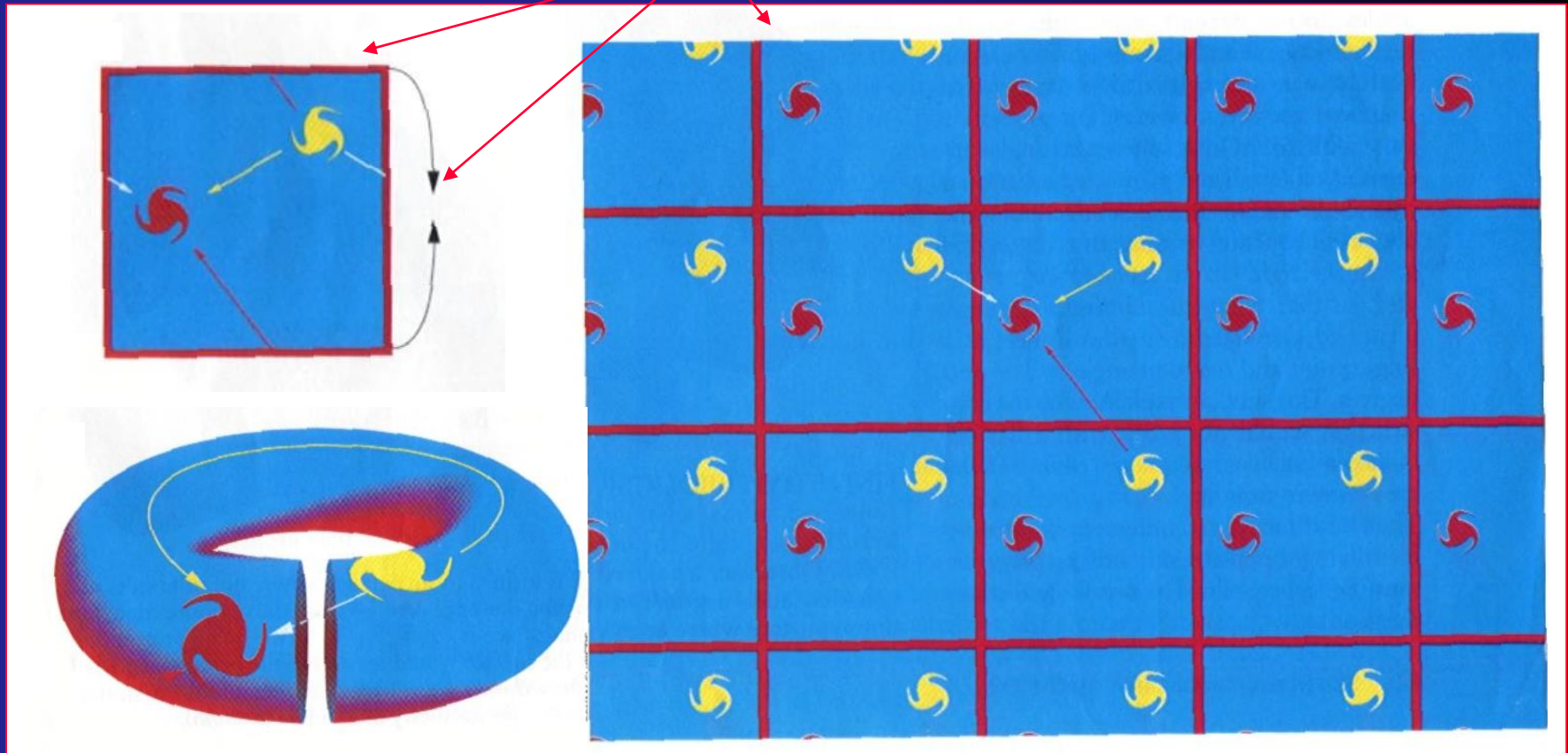
$$M = \frac{\tilde{M}}{\Gamma}, \quad \tilde{M} = (\mathbb{S}^3, \mathbb{H}^3, \mathbb{R}^3)$$



Where Γ is a freely acting, discrete group of isometries

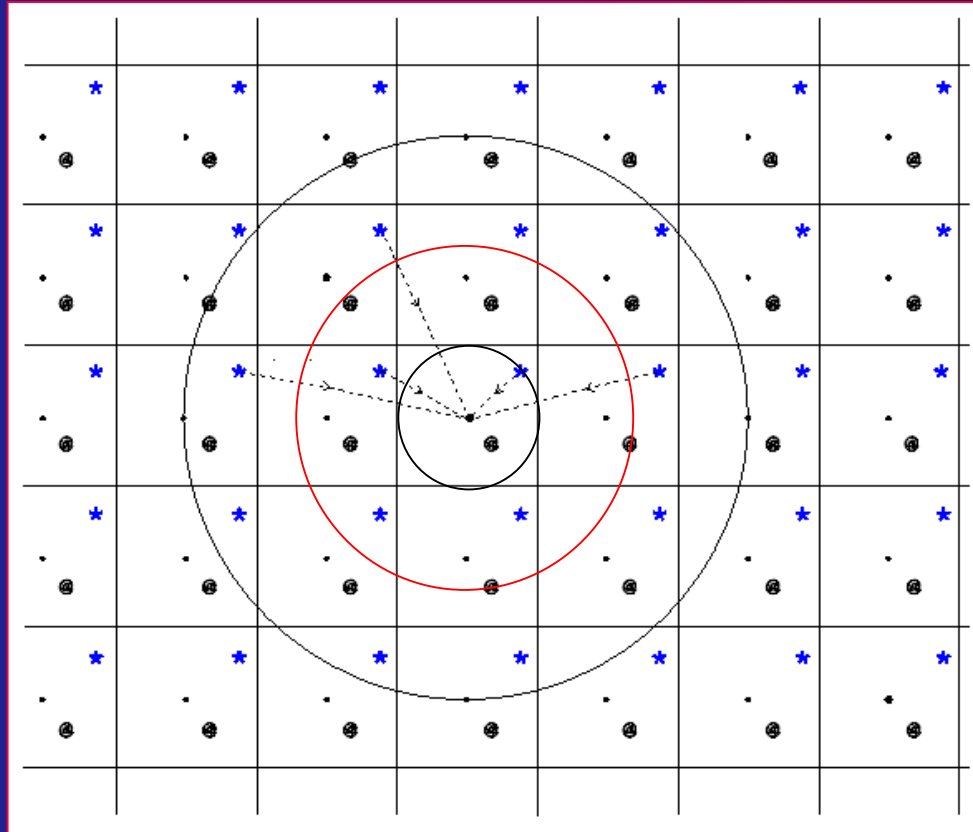
Detecting topology: multiple images

Covering space $(\mathbb{R}^2)$



Non-trivial topology $\Rightarrow$ Multiple images in covering space

Detectability: one holonomy at a time



$$\mathcal{M} = \frac{M}{\Gamma}, \quad M = R^3, \quad S^2 \text{ ou } H^3$$

- Each element in Γ generates a different image
- However, only images within the horizon are detectable
- γ is a Clifford Translation (CT) if $d(x, \gamma x) = cte.$ for any x

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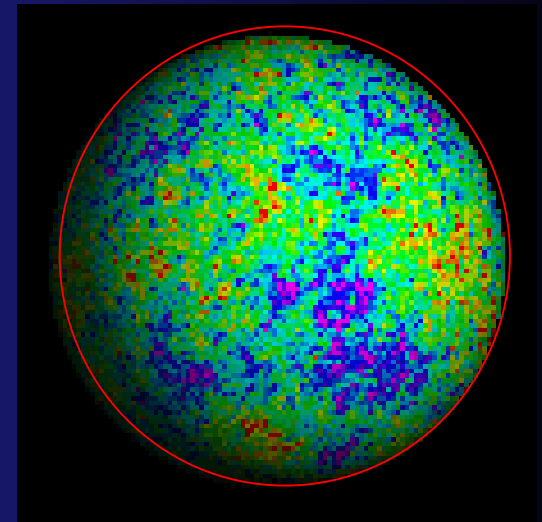
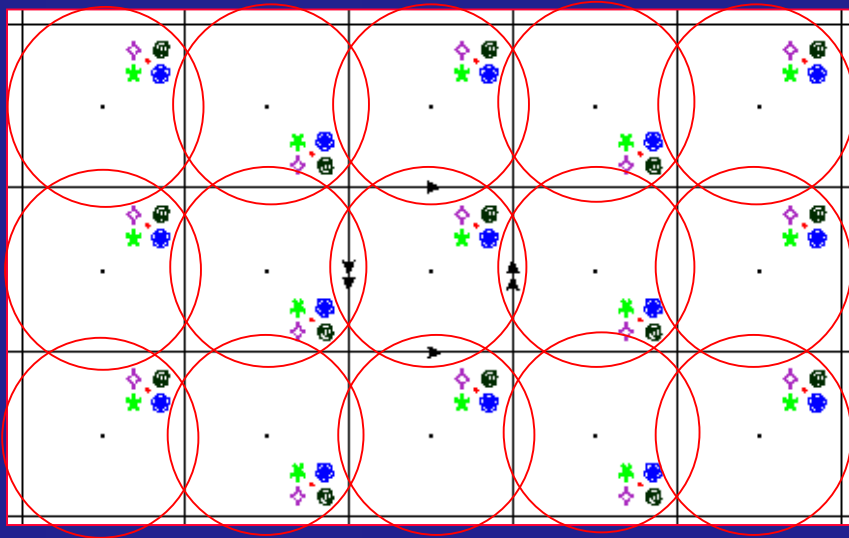
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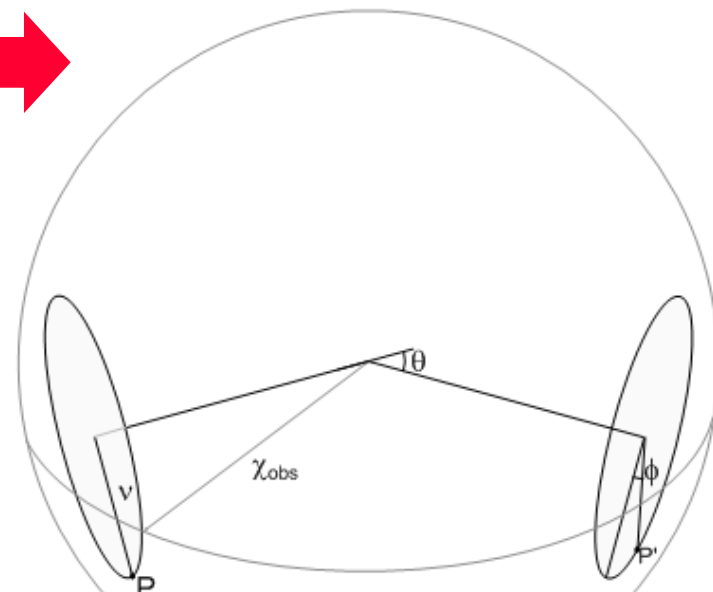
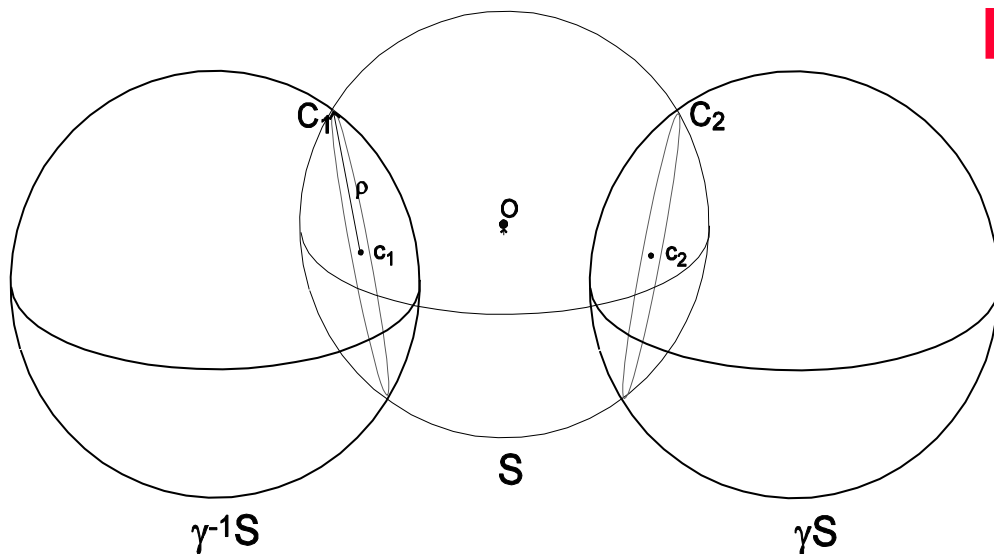
Circles in the sky

- In a universe with non-trivial topology, copies of the fundamental domain will tessellate the covering space
- If the topology is detectable, copies of the LSS will “spill over” and intersect along circles
- Along such intersecting circles temperature fluctuations will match



Circles in the sky

- A pair of circles in the CMB map with identical patterns of temperature fluctuations should be observable any detectable holonomy γ
- **If γ is a CT, the corresponding pair of circles will be antipodal**



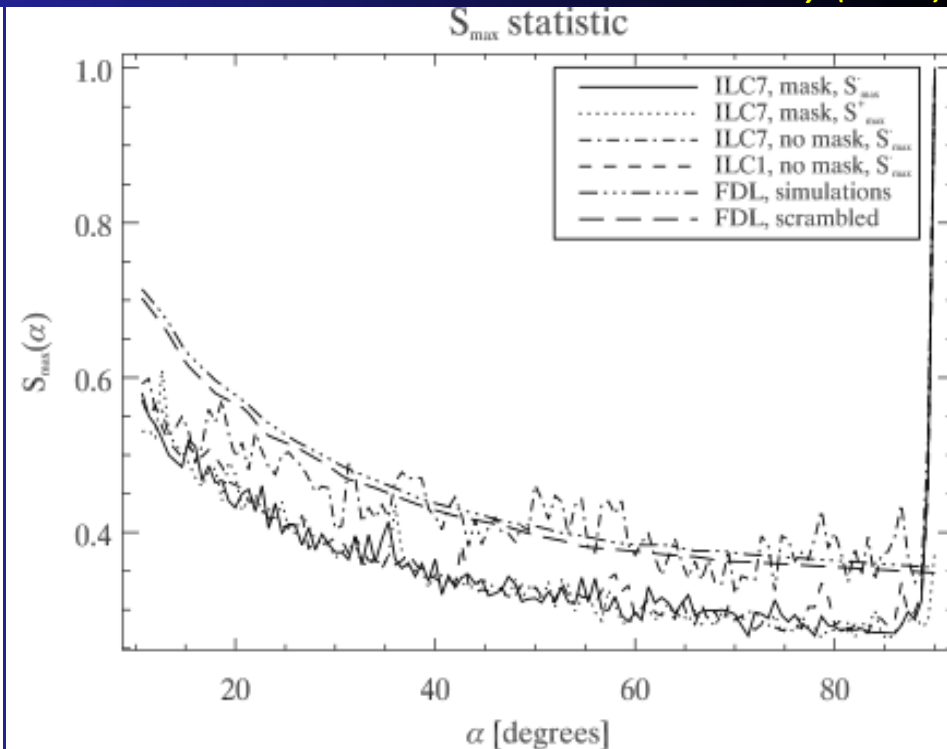
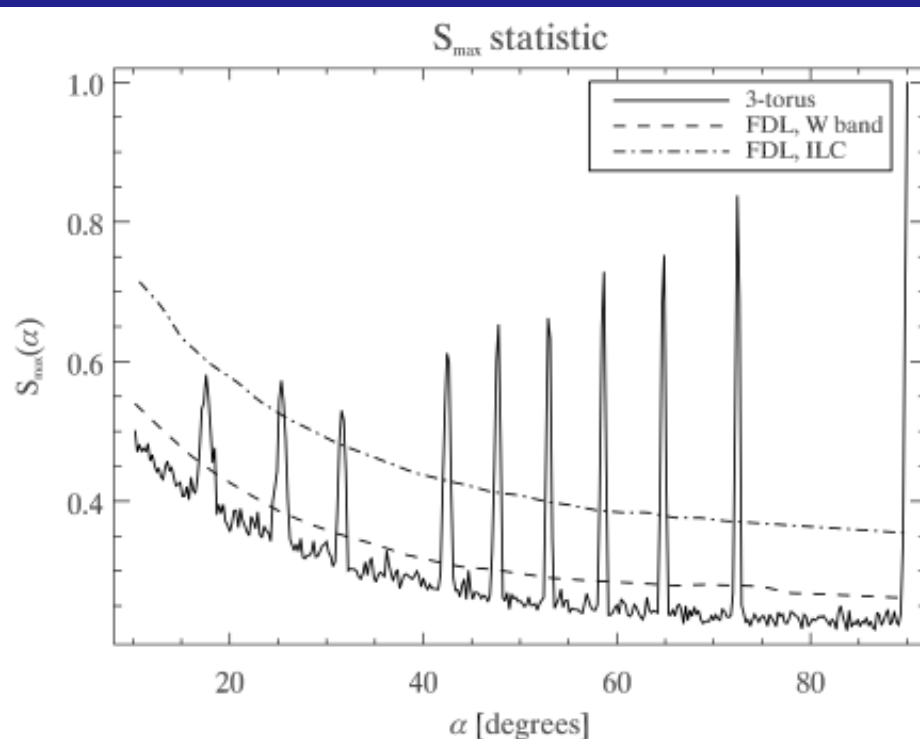
$$S(\varphi, \rho, \hat{c}_1, \hat{c}_2, \epsilon) = \frac{\langle 2T_1(\epsilon\phi)T_2(\phi + \varphi) \rangle}{\langle T_1(\phi)^2 + T_2(\phi + \varphi)^2 \rangle}$$

The $S_{\max}$ statistic

Proposed by Cornish, Spergel & Starkman in 1998 and modified (m-weighting) in 2004

$$S_{ij}(\alpha, \beta) = \frac{2 \sum_m m T_{im}(\alpha) T_{jm}^*(\alpha) e^{-im\beta}}{\sum_n n \left[|T_{in}(\alpha)|^2 + |T_{jn}(\alpha)|^2 \right]}$$

Bielewicz & Banday (2011)



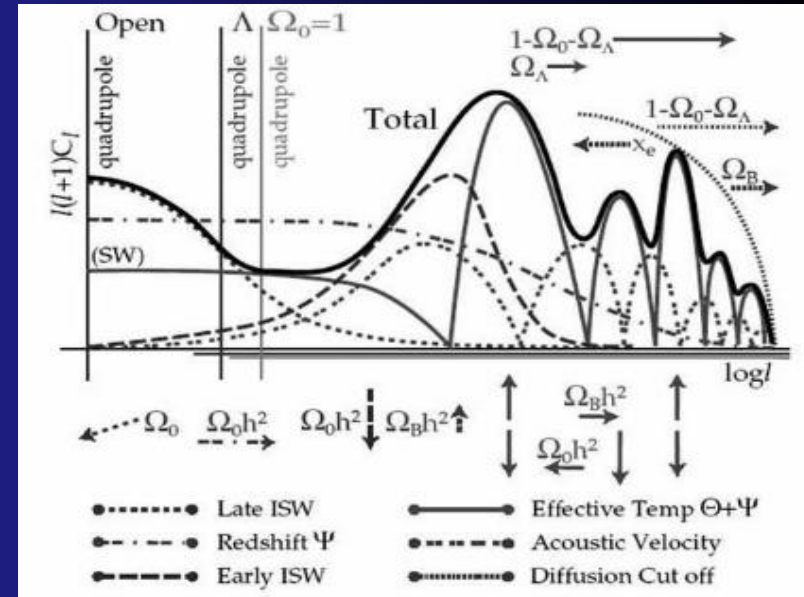
... but there are complications

Dominant:

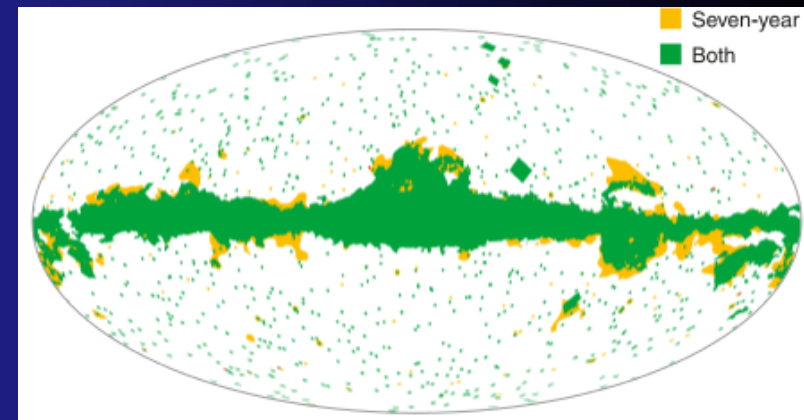
- ISW
- Doppler
- Foreground contamination

Sub-dominant:

- Last scattering shell thickness
- Beam profile
- Detector noise
- Proper movement
- Etc...



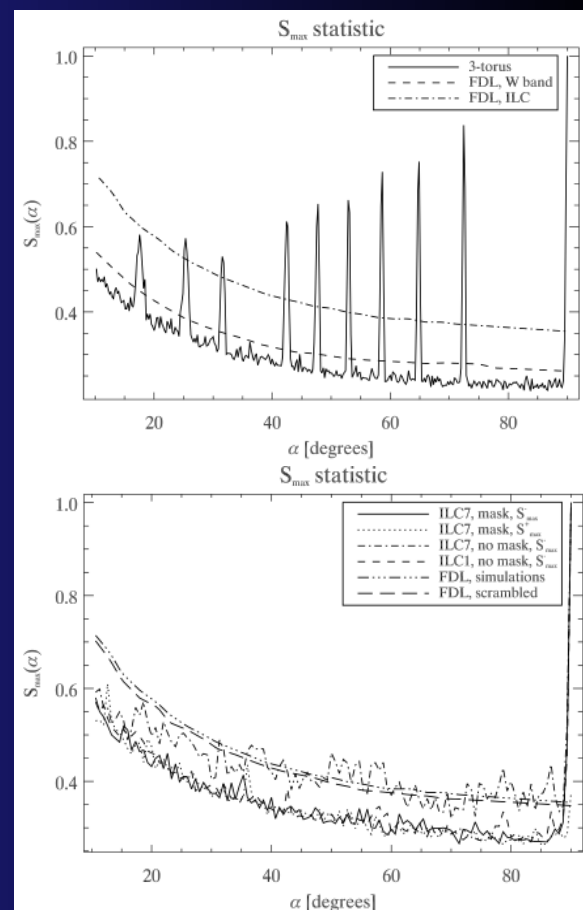
Hu, Sugiyama & Silk (1997)



KQ85, Gold et. al. (2010)

The searches: nothing (definitive) so far II

- Cornish et al (2004), Key et al (2007)
Nearly antipodal, WMAP 1y, ILC full sky
no signal
- Bielewicz & Banday, 2011
Antipodal only, WMAP 7y W band, KQ85y7 mask
no signal
- Vaudrevange et al, 2012
All pairs, WMAP 7y W band + ILC inside mask
weird signal, very likely foreground contamination
- Aurich & Lustig, 2013
Antipodal only, WMAP 7y W + V bands, KQ75 & KQ85 masks
marginal signal, likely foreground contamination



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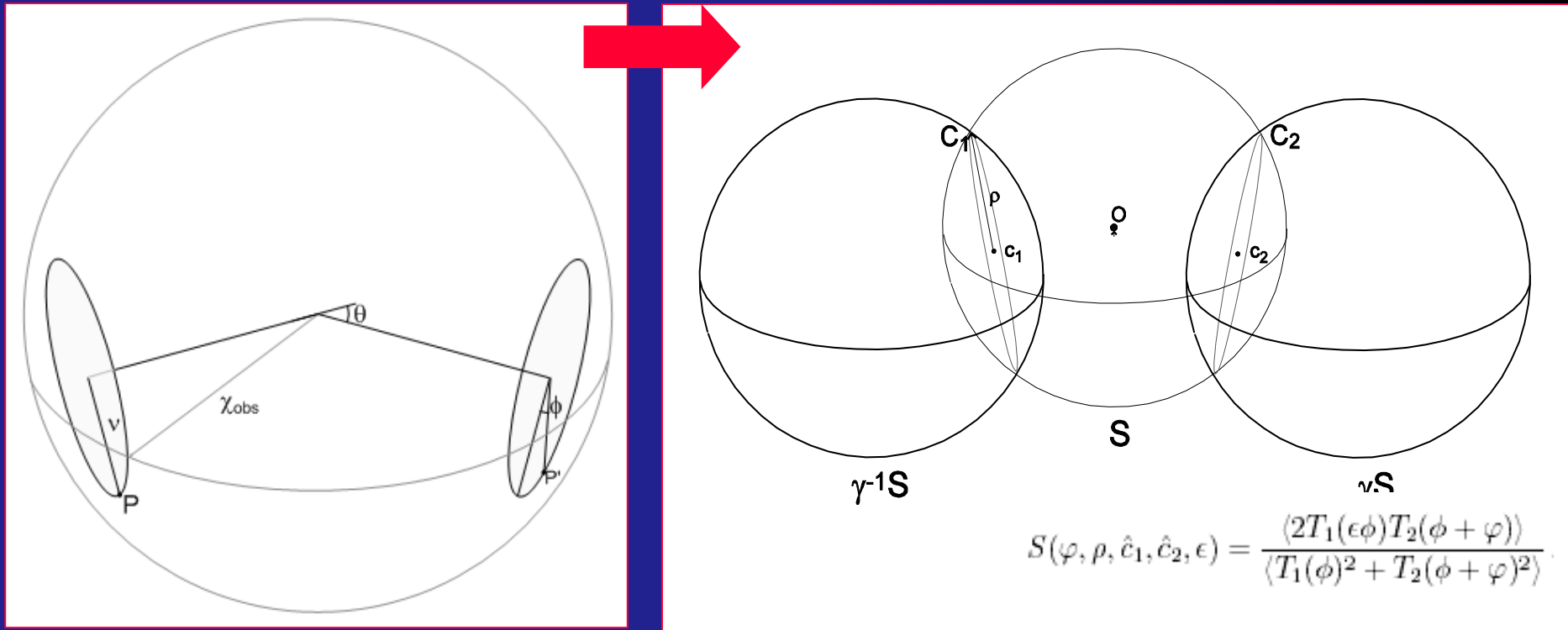
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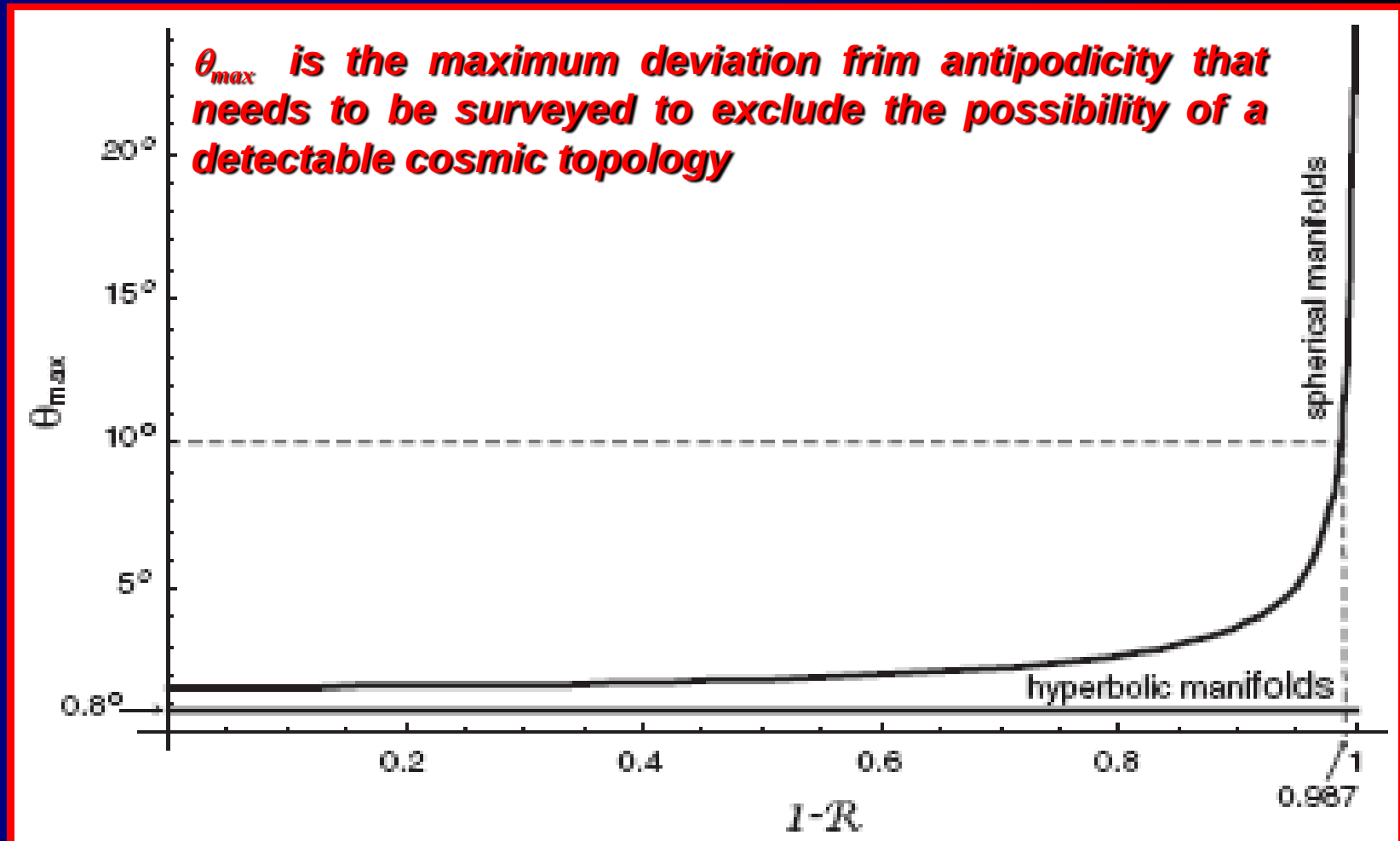
Determining the holonomy from a CitS detection

- One should be able to obtain holonomy parameters from the CitS parameters
- But is determining one holonomy enough to determine all the topology?
- **For non-translational flat holonomies, there is!**



(nearly) Excluding a detectable cosmic topology in the inflationary limit

- Fraction of observers R for which $\theta \leq \theta_{\max}$



(nearly) Excluding a detectable cosmic topology in the inflationary limit

- We can show that for small χ_{obs}

$$\theta \leq \frac{2 \cos \nu}{|z_2||z_1|} \chi_{obs} \quad (spherical)$$

$$\theta \leq 2 \cos \nu \chi_{obs} \quad (hyperbolical)$$

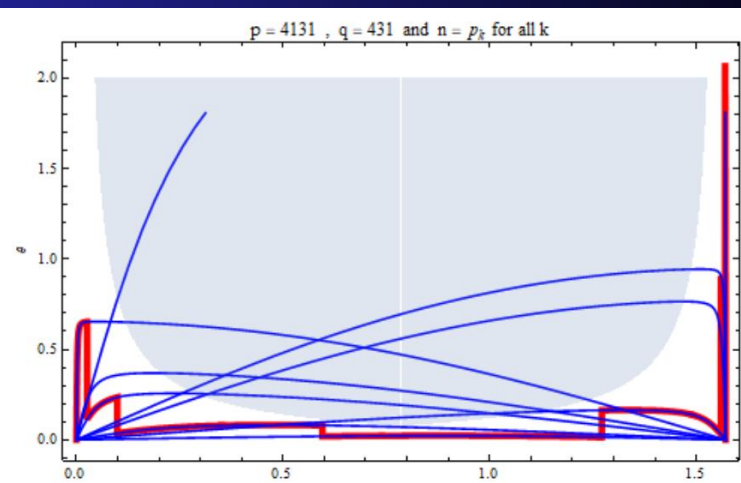
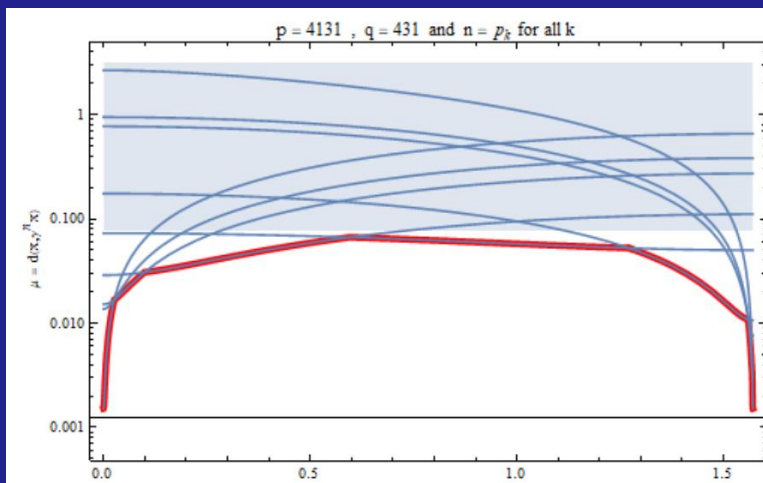
- **Yes, in the inflationary limit**

- For observers in hyperbolical universes, $\theta \leq 1.1^\circ$
- For 99% of observers in spherical universes, $\theta \leq 8^\circ$

Some potentially detectable lens spaces

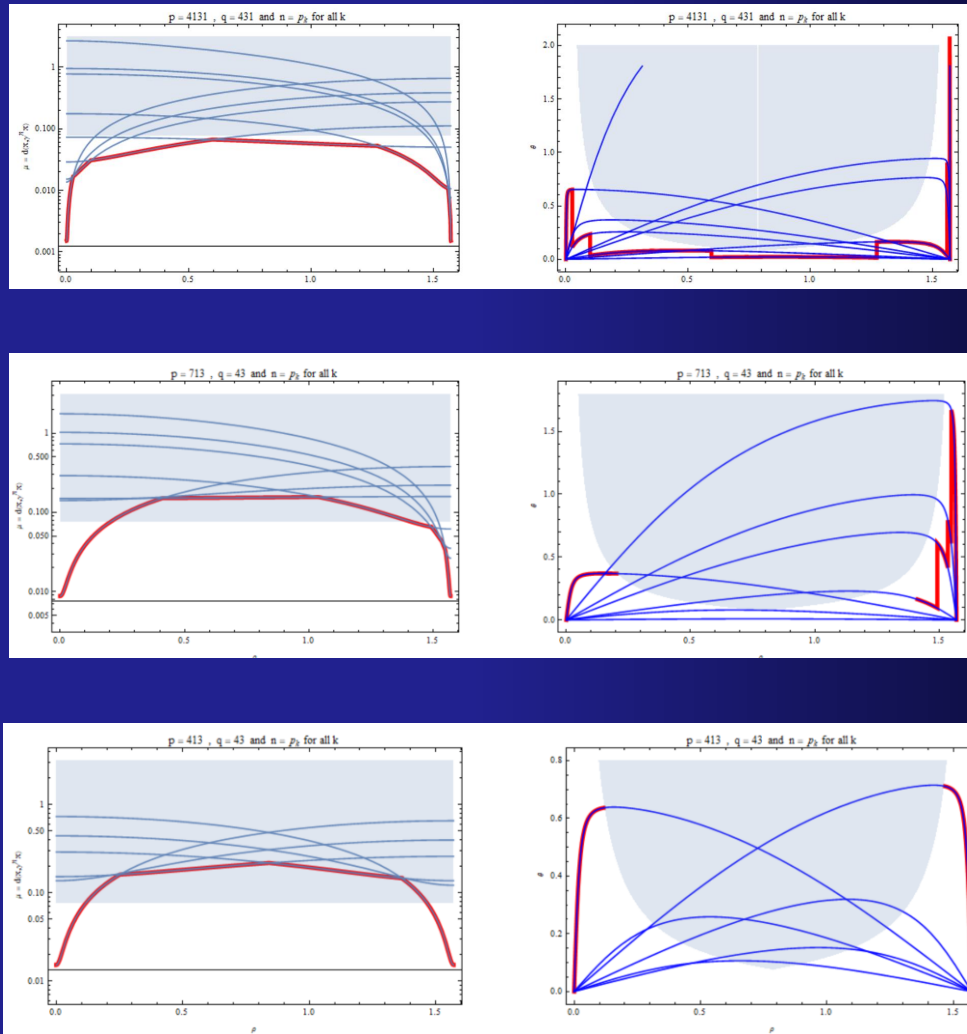
In this case, topology is detectable for all observers if $\chi > 0.05$

For most observers, the deviation from antipodality θ is *very small*



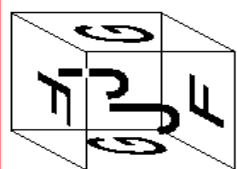
Some potentially detectable lens spaces

This is a general feature for detectable lens spaces with large p

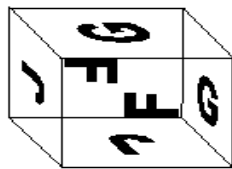


Deviation from antipodality for orientable flat manifolds

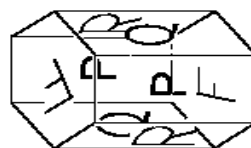
Symbol	Manifold	n	$\theta_{\max}$
E_1	three-torus	1,1,1	0°
E_2	half turn space	1,1,2	120°
E_3	quarter turn space	1,1,4	86°
E_4	third turn space	1,1,3	109°
E_5	sixth turn space	1,1,6	59°
E_6	Hantzsche-Wendt space	2,2,2	120°



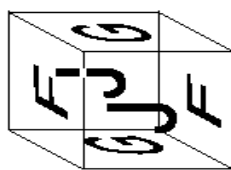
E2



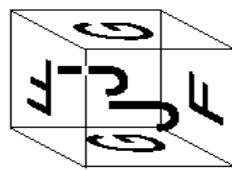
E6



E3



E1



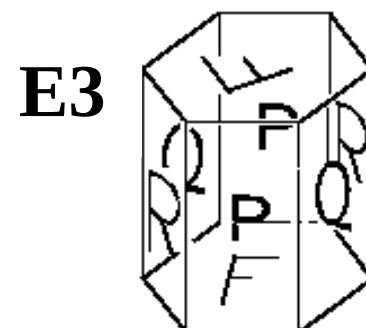
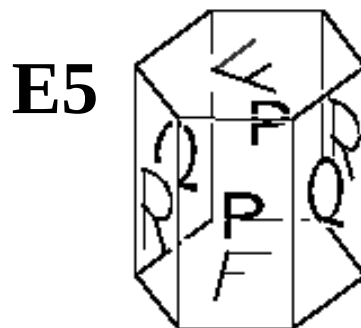
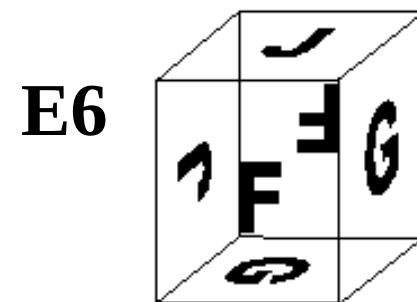
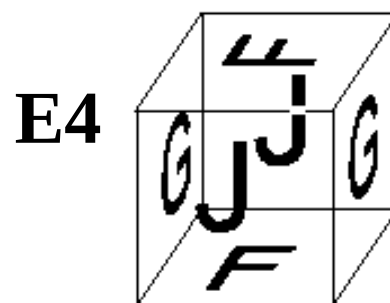
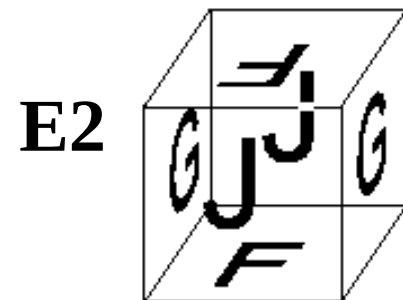
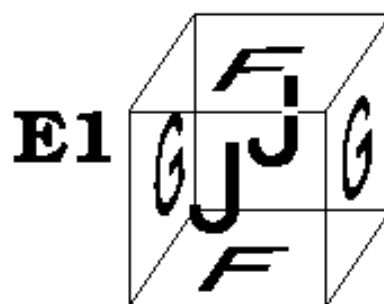
E4



E5

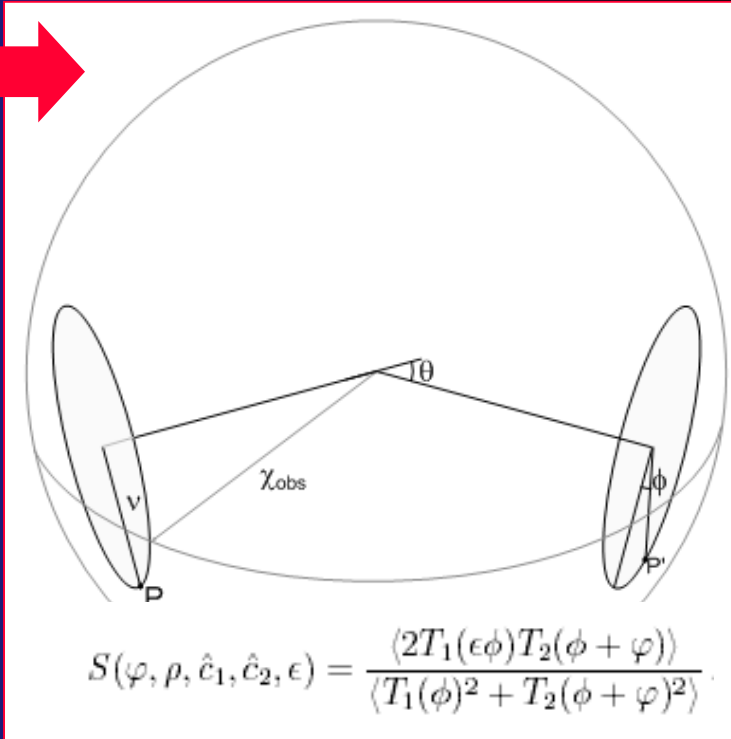
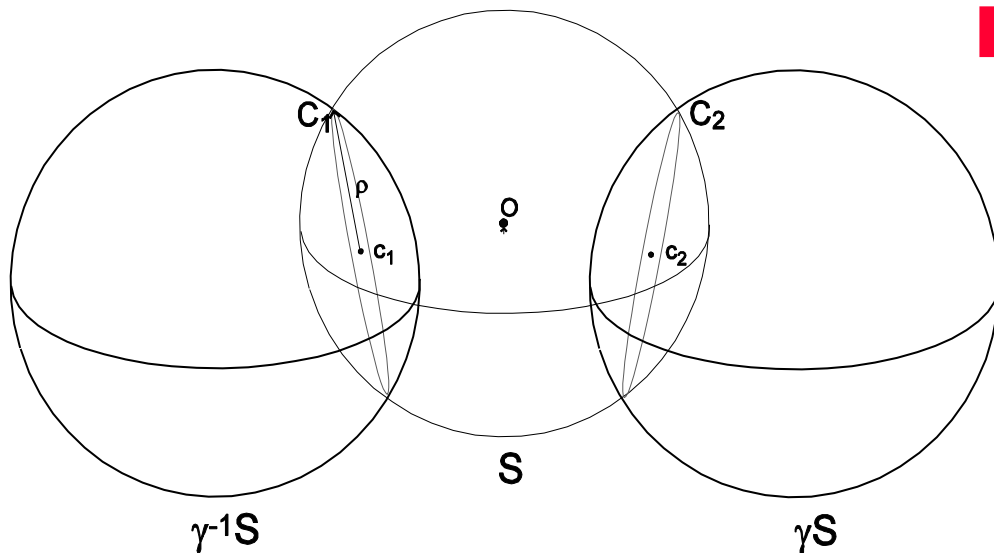
The (classes of) orientable flat 3-manifolds

- No characteristic scale
- Some parameters are free
- All but E_6 have as generators of Γ
 - 2 translations
 - 1 'screw motion'
 - $\alpha = 0, \pi, \pi/2, \pi/3, \pi/6$
- E_6 ('Hantzche-Wendt')
is generated by 3 screw motions
with $\alpha_i = \pi/2, i=1,2,3$



Determining CitS parameter from the holonomy

- It is possible to determine the CitS parameters (radius v , phase ϕ and deviation from antipodicity θ) from the holonomy parameters in both flat and spherical cases (Gomero 2005, 2013)



Determining the holonomy II

- The twist parameter of the holonomy can be expressed in terms of CitS parameters θ and ϕ

$$\cos \alpha = \frac{(\cos \phi + 1)(\cos \theta + 1)}{2} - 1$$

- The compactification length can also be expressed

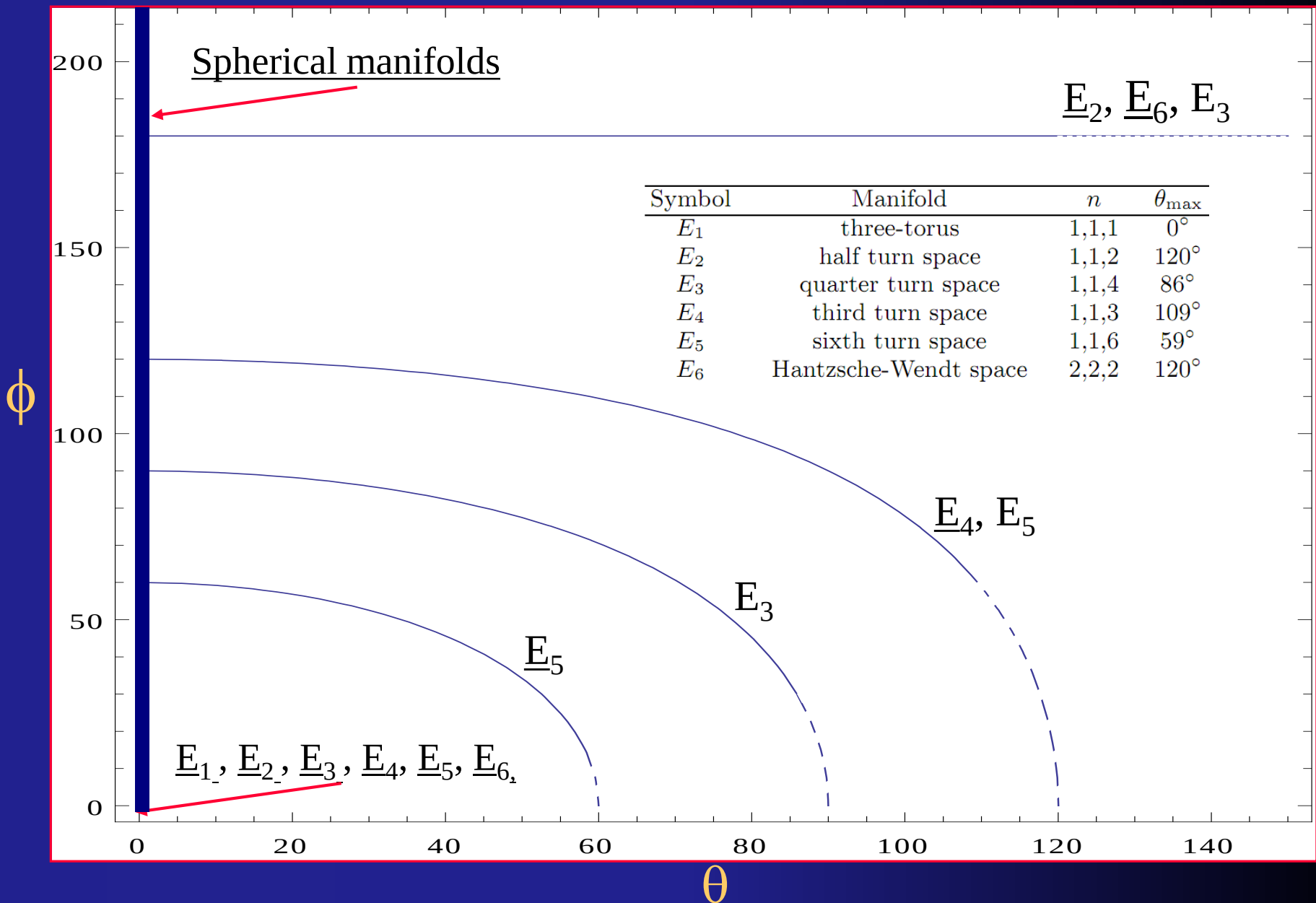
$$L = \chi_{obs} \cos \nu \sqrt{\frac{\cos \theta - \cos \alpha}{1 - \cos \alpha}}$$

L can take any value (non-rigidity), but α is constrained:

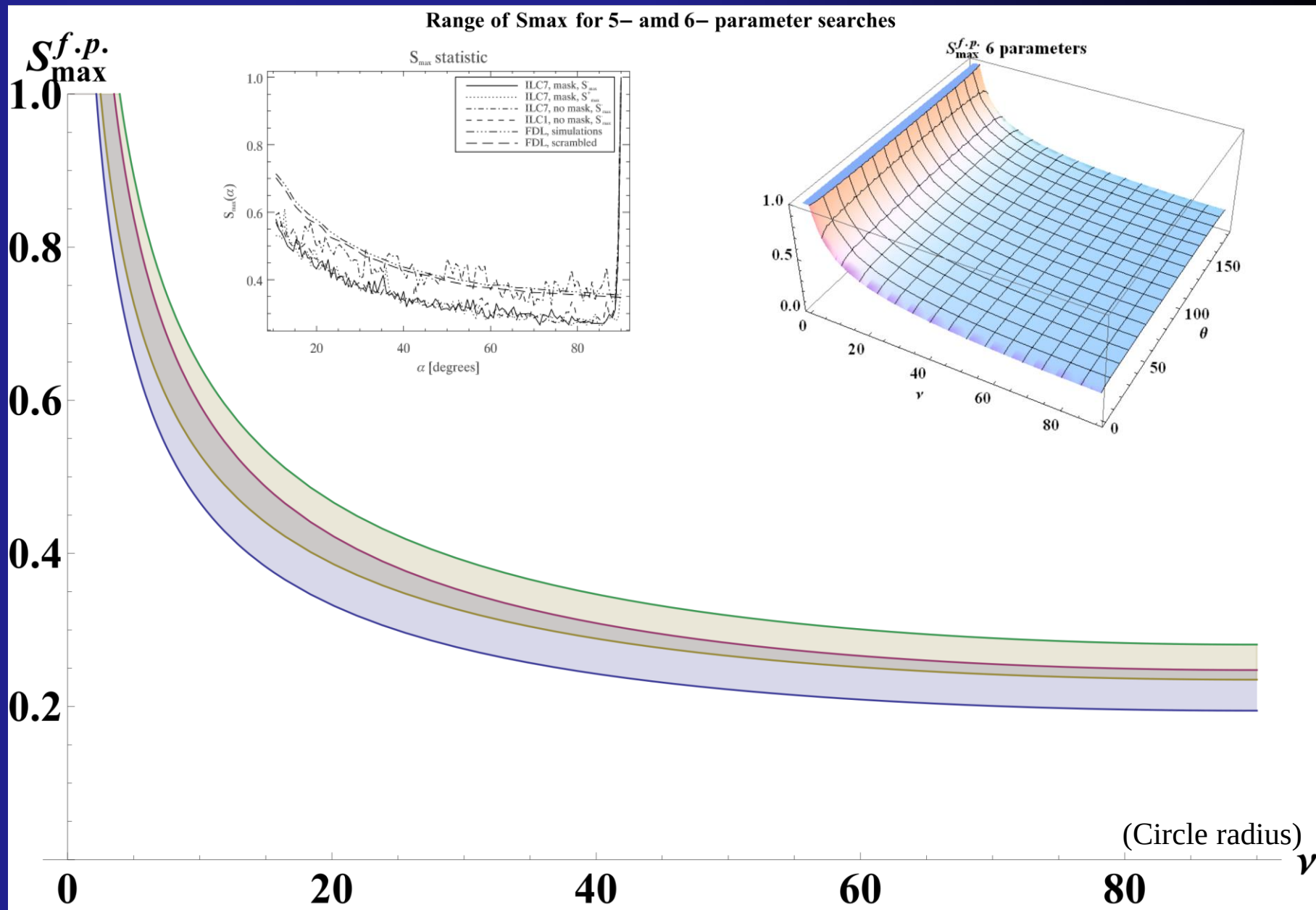
$$\alpha = 2\pi/n, \quad n = \{1, 2, 3, 4, 6\}$$

- In general more than one topology will include each given holonomy. However, only one of them will generate the shortest geodesic

Determining the topology from a single pair detection



A modest improvement in detectability with a focused search



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Where to look?

	Homogeneous	Non-homogeneous
Flat	• Antipodal circle pairs • No phase shift	• Non-antipodal pairs • Phase shift specified by deviation from antipodality
Spherical	• Antipodal pairs • Small Phase shift < 2.7° (Planck) < 0.004° (Inflation)	• Nearly antipodal pairs • Arbitrary phase

What can a pair detection tell us?

Phase = 0

Antipodal

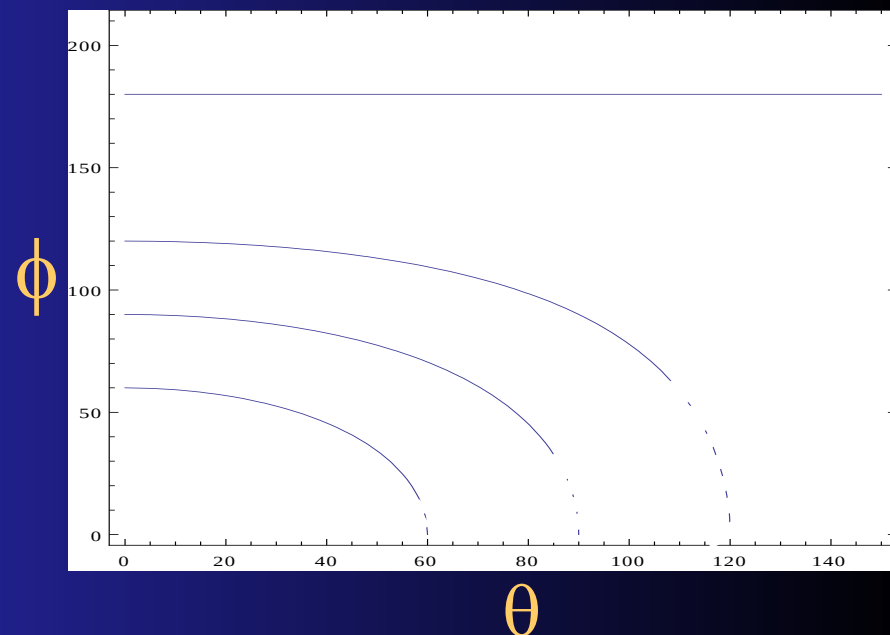
- It's a flat Universe!

Phase $\neq 0$

- It's a positively-curved Universe!
- Universe is globally homogeneous
- With sufficient precision, value of Ω_k

Non-antipodal

- Universe is not globally homogeneous
- Either flat or positively-curved, depending on values of ϕ and θ
- In many cases, can fully specify topology and our position



The way forward

- The issue of a detectable cosmic topology is not settled!
- Focused searches:
 - Computationally more efficient
 - Reduces false-positive threshold
- Systematically look for matching partial circles (arcs)
- Look for topology in polarization maps? (cross-validation)

